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ELASTIC LOADING OF THICK-WALLED HIGH PRESSURE CYLINDERS

by R. EPAIN and B. VODAR

We consider a thick-walled cylinder submitted to uniformly distributed internal and external pressures and a uniformly distributed longitudinal load and we establish the relations between these loads such that the cylinder does not undergo plastic deformation. For this purpose we use the criteria of Von Mises and of Tresca as well as a linearized form of the criterium of the intrinsic curve of Mohr-Caquot. We describe a graphic method which allows the resolution of these problems in a more varied manner than that of the calculations. Wr finish with several remarks on the conditions and limits in the use of this method.

A hollow cylinder of circular cross-section (figure 1) is submitted to internal p_i , external p_e and longitudinal p_ℓ uniformly distributed pressures, and the limiting relations, i e , the limit in which the vessel undergoes no plastic deformation, between these quantities are established. These relations will be referred to as elastic loading conditions. (X)

(X) Throughout this paper we will use "conditions of elastic loading" for the French "condition de portance élastique". A better expression might be "elastic limit load".

We will establish these relations for three criteria of plasticity: the criterium of Von Mises [1] that of Mohr-Caquot [2] and finally, that of Tresca [3].

The well known formulas of Lamé give the radial, σ_r, circumferential σ_θ, and longitudinal σ_z, stresses as functions of p_i, p_l and p_e in the following form:

$$\sigma_r = p_i \frac{1}{k^2-1}\left[1-\frac{r_i^2}{r^2}k^2\right] + p_e \frac{k^2}{k^2-1}\left[\frac{r_i^2}{r^2}-1\right],$$

$$\sigma_\theta = p_i \frac{1}{k^2-1}\left[1+\frac{r_i^2}{r^2}k^2\right] - p_e \frac{k^2}{k^2-1}\left[\frac{r_i^2}{r^2}+1\right],$$

$$\sigma_z = -p_l,$$

where $k = \frac{r_e}{r_i}$ is the ratio of external radius to the internal radius of the cylinder. Substituting these expressions into the corresponding criteria we obtain the desired conditions of elastic loading.

References

1 Mechanik der festen Körper im plastisch deformablen Zustand - Von Mises, R, Göttinger Nachrichten 1913.

2 Définition du domaine élastique dans les corps isotropes-Caquot, A, Proc. of the 4 th Congr. Intern. Appl. Mech. Cambridge 1935, p. 24.

3 Mémoire sur l'écoulement des corps solides. Tresca - Mémoires présentés par divers savants 1868, volume 18, pages 773 à 799.

4 Mémoires sur l'équilibre intérieur des corps solides homogènes-Lamé & Clapeyron Mémoires présentés par divers savants 1833.

1) **Relations of elastic loading for the criterium of Von Mises** [5]

On expressing the principal stresses as functions of the pressures p_i, p_ℓ, p_e in the relation of Von Mises, given by $(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 < 2\,\sigma_0^2$; we obtain the condition of elastic loading corresponding to this criterium:

$$\frac{2}{(k^2-1)^2}\Big[3k^4\frac{r_i^4}{r^4}(p_i - p_e)^2 + p_i^2 + k^4 p_e^2 - 2k^2 p_i p_e + p_\ell^2(k^2-1)^2 + 2p_i p_\ell(k^2-1) - 2p_e p_\ell k^2(k^2-1)\Big] < 2\,\sigma_0^2;$$

where σ_0 is the elastic limit of the material for pure tension. The left hand member of this expression being maximum for $r = r_i$ it is seen that plastic deformations will occur, either first at the internal diameter of the cylinder whatever might be the relative values of p_i and p_e, or simultaneously in the entire thickness of the cylinder for the particular case $p_i = p_e = 0$ and $p_\ell = \pm\,\sigma_0$.

If we now take the case where a plastic deformation is possible, we should write $r = r_i$ and the inequality in the above equation becomes equality. On the pressure space p_i, p_ℓ, p_e, the surface described by this equality is an elliptic cylinder with its axis pointing in the direction $(1, 1, 1,)$. The elliptic cross-section varies both in dimension and orientation with ratio $k = \frac{r_e}{r_i}$. This surface has meaning only as long as p_i and p_e are positive, while $p_\ell = -\frac{L}{\pi(r_e^2 - r_i^2)}$, may be positive, negative or vanishing, depending on the value and sign (tension or compression) of the longitudinal load L.

In order to study this surface, we transfer from the system of axes p_i, p_ℓ, p_e, to the system V, W, Z (both are orthonormal), V and W being respectively, coincident with the minor and major axes of the ellipse of the normal cross-section, while Z, parallel to the generating line of the elliptic cylinder is inclined at equal angles to p_i, p_ℓ, p_e.

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(5) Contribution à l'étude de la résistance des cylindres épais elasto-plastiques – EPAIN R. Thèse Doct. d'Université Paris 1961

. The dimensions of the ellipse of the normal cross-section are given by :

$$\frac{\text{minor axis}}{2} = \frac{\sigma_0 (M-1)}{\sqrt{4M^2 - M + 1 + \sqrt{7M^4 + 10M^3 - 2M + 1}}}$$

$$\frac{\text{major axis}}{2} = \frac{\sigma_0 (M-1)}{\sqrt{4M^2 - M + 1 - \sqrt{7M^4 + 10M^3 - 2M + 1}}}$$

while its orientation relative to the projections P'_i, P'_ℓ, P'_e of the axes P_i, P_ℓ, P_e on the plane π perpendicular to Z is determined by the following relations :

$$\operatorname{tg} \psi = \sqrt{\frac{(V_1/V_3)^2 + (V_2/V_3)^2 + 1}{(W_1/W_3)^2 + (W_2/W_3)^2 + 1}}, \qquad \text{with } \psi = \text{angle } V, P'_e,$$

where
$$V_1/V_3 = \frac{3M^2(M-1) + X_1}{3M^2(M-1) - MX_1}, \qquad W_1/W_3 = \frac{3M^2(M-1) + X_2}{3M^2(M-1) - MX_2},$$

$$V_2/V_3 = \frac{3M^2(M-1)^2 + (M-1)X_1}{3M^2(M-1)^2 - (3M+1)MX_1}, \qquad W_2/W_3 = \frac{3M^2(M-1)^2 + (M-1)X_2}{3M^2(M-1)^2 - (3M+1)MX_2},$$

$$\left.\begin{matrix} X_1 \\ X_2 \end{matrix}\right| = 4M^2 - M + 1 \pm \sqrt{7M^4 + 10M^3 - 2M + 1}, \qquad M = k^2.$$

For k = 1, corresponding to a hypothetical cylinder of zero thickness, the ellipse is reduced to a line (zero surface) along P'_ℓ. For $k = \infty$ corresponding to a cylinder of infinite thickness or to a capillary tube, the ellipse has the dimensions indicated in the figure 2.

Now, if in the pressure space we represent the load by the vector $\overrightarrow{OP}$ with components p_i, p_e, p_e, we can make the following remarks :

1) plastic flow is only possible if P lies on the elliptic cylinder;

2) since the hydrostatic load is represented by a vector parallel to Z, only the component of $\overrightarrow{OP}$ in the plane π is necessary for determining whether the material does or does not remain elastic.

3) The ellipse corresponding to $k = \infty$ having a finite dimension confirms the known result that a finite state of load is sufficient to create a plastic deformation in a cylinder od infinite thickness.

The second remark leads to the establishment of a graphic method permitting the resolution of two types of problems relative to elastic loading. On the first graph A three equidistant axes p_i', p_e', p_e' are traced as well as the axes V for different values of K. For these same values one traces on transparent paper a series of graphs B representing the corresponding ellipses. The number of these graphs is limited both by the allowed interpolations and the fact that k = 4 constitutes a limiting value in practice. Finally one obtains a new simplification by scaling the designs to $\sqrt{3/2}$ and on letting $\sigma_0 = 1$.

The first problem is the following : " given k and σ_0 determine whether this cylinder remains elastic under the loads p_i, p_e, p_e " . One superimposes the graphs A en B making the axes V coincide and one traces the projection of $\overrightarrow{OP}$ on the plane π whose components onto p_i', p_e', p_e' are respectively p_i/σ_0, p_e/σ_0, p_e/σ_0. The cylinder does or does not remain elastic according to whether P falls inside (see figure 3) or on the ellipse.

In particular one finds the following well-known result : the maximum internal pressure p_i that a cylinder can withstand elastically in the case where p_e is zero is equal to :

$\frac{\sigma_0}{\sqrt{3}}\left(1-1/k^2\right)$ for a closed cylinder,

$\frac{\sigma_0}{\sqrt{3}}\left(1-1/k^2\right)/\sqrt{1+1/3k^4}$ for an open cylinder,

$\frac{\sigma_0}{\sqrt{3}}\left(1-1/k^2\right)/\sqrt{1+(1-2\nu)^2/3k^4}$ for a plane strain condition.

The second type of problem can be stated as " given a cylinder characterized by k and σ_0 and submitted to an external pressure p_e determine p_ℓ such that p_i will be maximum and find this value". One begins as before, then one traces $OC = p_e/\sigma_0$ and Δ paralled to p'_ℓ and tangent to the ellipse. One then deduces that $p_\ell = \sigma_0$ CD and $p_i = \sigma_0$ DP. (see figure 4).

The determination of the extrema ca be done by the preceding method and this allows one to obtain the results shown in the table below.

MAXIMUM VALUE	GIVEN VALUE	CORRESPONDING VALUE
$p_i = p_e + \frac{\sigma_0}{\sqrt{3}}\left(1 - \frac{1}{k^2}\right)$	p_e	$p_\ell = p_e - \frac{\sigma_0}{\sqrt{3}k^2}$
$p_i = \frac{2\sigma_0}{\sqrt{3}} + p_\ell$	p_ℓ	$p_e = \frac{\sigma_0}{2\sqrt{3}}\left(3 + \frac{1}{k^2}\right) + p_\ell$
$p_e = p_e + \sigma_0\sqrt{1 + 1/3k^4}$	p_e	$p_i = p_e + \frac{\sigma_0}{\sqrt{3}}\frac{3k^2+1}{\sqrt{3k^4+1}}$
$p_e = p_i + \frac{\sigma_0}{\sqrt{3}}\left(1 - \frac{1}{k^2}\right)$	p_i	$p_\ell = p_i + \frac{\sigma_0}{\sqrt{3}}$
$p_\ell = p_i + \frac{2\sigma_0}{\sqrt{3}}$	p_i	$p_e = p_i + \frac{\sigma_0}{2\sqrt{3}}\left(1 - \frac{1}{k^2}\right)$
$p_\ell = p_e + \sigma_0\sqrt{1 + 1/3k^4}$	p_e	$p_i = p_e - \frac{\sigma_0}{\sqrt{3}}\left(1 - \frac{1}{k^2}\right) \Big/ \sqrt{3k^4+1}$

2 - RELATIONS OF ELASTIC LOADING CORRESPONDING TO THE CRITERIUM OF THE INTRINSIC CURVE OF MOHR-CAQUOT -

For simplification we utilize a linearized intrinsic curve obtained by drawing the tangents D to the circles of diameters σ_o and σ_c where σ_o and σ_c are the absolute values of the elastic limits for pure tension and pure compression (see figure 5).

There is plastic flow at a point in the wall of the cylinder if the local values of the constraints are such that the circle of Mohr constructed by the major σ_M and the minor σ_m stresses is tangent to or cuts the lines D .

In the limiting case where this circle is tangent to the lines D the figure 5 shows that one has :

radius of the Mohr circle = $\dfrac{\sigma_M - \sigma_m}{2} = b\cos\alpha - \dfrac{\sigma_M + \sigma_m}{2}\sin\alpha$,

where $\dfrac{\sigma_M + \sigma_m}{2}$ represents the abscissa of the center of this circle ,

while $\tau = \mp \sigma \operatorname{tg}\alpha \pm b$ are the equations representing the tangents D.

On noting that one can write $\sin\alpha = \dfrac{\sigma_c - \sigma_o}{\sigma_o + \sigma_c}$ and $b\cos\alpha = \dfrac{\sigma_o \sigma_c}{\sigma_o + \sigma_c}$ in the preceding equation, the necessary condition that the cylinder remains elastic is expressed by the inequality

$$\sigma_M - \sigma_m \frac{\sigma_o}{\sigma_c} < \sigma_o .$$

According to the relative magnitudes of the principal stresses this equation is written in six different ways :

$$\sigma_\theta - \frac{\sigma_o}{\sigma_c}\sigma_z < \sigma_o , \quad \text{for } \sigma_\theta > \sigma_r > \sigma_z ,$$

$$\sigma_\theta - \frac{\sigma_o}{\sigma_c}\sigma_r < \sigma_o , \quad \text{for } \sigma_\theta > \sigma_z > \sigma_r ,$$

$$\sigma_z - \frac{\sigma_o}{\sigma_c}\sigma_r < \sigma_o , \quad \text{for } \sigma_z > \sigma_\theta > \sigma_r ,$$

the three other forms being obtained by permutation of σ_M and σ_m .

When one expresses the principal stresses as a function of the loads in the three preceding inequalities there appears on the left hand side the expression $\frac{r_e^2}{r^2}\,\frac{p_i - p_e}{k^2-1}$, which is always positive since $\sigma_\theta > \sigma_r$ for these three cases. The result of this circumstance is that these expressions become maximum for $r = r_i$. This signifies that, just as for the criterium of Von Mises, plastic flow begins at the internal diameter.

In the limit and with $r = r_i$ these inequalities above are written in the following form :

$$p_i \frac{1+k^2}{k^2-1} - p_e \frac{2k^2}{k^2-1} + \frac{\sigma_0}{\sigma_c} p_\ell = \sigma_0 \qquad \text{for } \sigma_\theta > \sigma_r > \sigma_z$$

$$p_i \left[\frac{1-\sigma_0/\sigma_c + k^2(1+\sigma_0/\sigma_c)}{2k^2}\right] - p_e = \sigma_0 \frac{k^2-1}{2k^2} \qquad \text{for } \sigma_\theta > \sigma_z > \sigma_r$$

$$-p_\ell + \frac{\sigma_0}{\sigma_c} p_i = \sigma_0 \qquad \text{for } \sigma_z > \sigma_\theta > \sigma_r$$

$$-p_i \frac{\sigma_0}{\sigma_c} \frac{1+k^2}{k^2-1} + p_e \frac{\sigma_0}{\sigma_c} \frac{2k^2}{k^2-1} - p_\ell = \sigma_0 \qquad \text{for } \sigma_z > \sigma_r > \sigma_\theta$$

$$p_i \left[\frac{1-\sigma_0/\sigma_c - k^2(1+\sigma_0/\sigma_c)}{2k^2\,\sigma_0/\sigma_c}\right] + p_e = \sigma_0 \frac{k^2-1}{2k^2\,\sigma_0/\sigma_c} \qquad \text{for } \sigma_r > \sigma_z > \sigma_\theta$$

$$-p_i + \frac{\sigma_0}{\sigma_c} p_\ell = \sigma_0 \qquad \text{for } \sigma_r > \sigma_\theta > \sigma_z$$

The equations represent six planes in the space p_i, p_ℓ, p_e.

We now seek to determine the contours formed by their traces on the planes perpendicular to the line $p_i = p_\ell = p_e$.
In order to do this we will utilize a new coordinate system V, W, Z where the axis Z coincides with the line $p_i = p_\ell = p_e$ while W is at the intersection of the plane formed by the coordinates p_i and p_e and the plane π perpendicular to Z and passing through the origin. Under these conditions the new orthonormal vectors are written as functions of the old coordinate system in the following way :

$$\vec{V} = -1/\sqrt{6}\,\vec{p_i} + 2/\sqrt{6}\,\vec{p_\ell} - 1/\sqrt{6}\,\vec{p_e}$$

$$\vec{W} = -1/\sqrt{2}\,\vec{p_i} + 1/\sqrt{2}\,\vec{p_e}$$

$$\vec{Z} = 1/\sqrt{3}\,\vec{p_i} + 1/\sqrt{3}\,\vec{p_\ell} + \frac{1}{\sqrt{3}}\vec{p_e}$$

Inversely the old orthonormal vectors $\vec{p_i}$, $\vec{p_\ell}$ and $\vec{p_e}$ are given in the new system by :

$$\vec{p_i} = -1/\sqrt{6}\,\vec{V} - 1/\sqrt{2}\,\vec{W} + 1/\sqrt{3}\,\vec{Z}$$

$$\vec{p_\ell} = 2/\sqrt{6}\,\vec{V} + 1/\sqrt{3}\,\vec{Z}$$

$$\vec{p_e} = -1/\sqrt{6}\,\vec{V} + 1/\sqrt{2}\,\vec{W} + 1/\sqrt{3}\,\vec{Z} .$$

It follows that the equations of the six planes described above are written in the new coordinate system as indicated below (for $\sigma_o \neq \sigma_c$):

$$(1)\; V = \frac{\sqrt{3}}{1+2\sigma_o/\sigma_c}\,\frac{3k^2+1}{k^2-1}\,W + \frac{1-\sigma_o/\sigma_c}{1+2\sigma_o/\sigma_c}\sqrt{2}\,Z + \frac{\sigma_o\sqrt{6}}{1+2\sigma_o/\sigma_c} ,$$

$$(2)\; V = \frac{\sqrt{3}}{1-\sigma_o/\sigma_c}\left[\frac{3k^2+1}{k^2-1} + \frac{\sigma_o}{\sigma_c}\right]W + \sqrt{2}\,Z + \frac{\sigma_o\sqrt{6}}{1-\sigma_o/\sigma_c} ,$$

$$(3)\; V = -\frac{\sigma_o/\sigma_c\,\sqrt{3}}{2+\sigma_o/\sigma_c}\,W - \frac{1-\sigma_o/\sigma_c}{2+\sigma_o/\sigma_c}\sqrt{2}\,Z - \frac{\sigma_o\sqrt{6}}{2+\sigma_o/\sigma_c} ,$$

$$(1')\; V = \frac{\sqrt{3}\,\sigma_o/\sigma_c}{2+\sigma_o/\sigma_c}\,\frac{3k^2+1}{k^2-1}\,W - \frac{1-\sigma_o/\sigma_c}{2+\sigma_o/\sigma_c}\sqrt{2}\,Z - \frac{\sigma_o\sqrt{6}}{2+\sigma_o/\sigma_c} ,$$

$$(2')\; V = -\frac{\sqrt{3}}{1-\sigma_o/\sigma_c}\left[\frac{3k^2+1}{k^2-1}\,\frac{\sigma_o}{\sigma_c} + 1\right]W + \sqrt{2}\,Z + \frac{\sigma_o\sqrt{6}}{1-\sigma_o/\sigma_c} ,$$

$$(3')\; V = -\frac{\sqrt{3}}{1+2\sigma_o/\sigma_c}\,W + \frac{1-\sigma_o/\sigma_c}{1+2\sigma_o/\sigma_c}\sqrt{2}\,Z + \frac{\sigma_o\sqrt{6}}{1+2\sigma_o/\sigma_c} .$$

Now on cutting these planes by the plane Z = constant we obtain six lines which form the desired contour.

One notes that the slopes of these lines vary with the ratio σ_o/σ_c

and $k = \frac{\tau_e}{\tau_i}$ (except for the lines 3 and 3').

Further their ordinate intersections are functions of σ_0/σ_c and in particular, of $Z = \frac{1}{\sqrt{3}}(P_i + P_\ell + P_e)$. This signifies that, contrary to the criterium of Mises, the extent of the elastic domain in the present case is no longer independant of the hydrostatic component of the load vector. One can further note that with the system of axes V, W, Z, the V axis coincides with the projection P_ℓ' of P_ℓ onto the plane π. The figure 6 shows how one can easily trace the contour corresponding to given values of the constants σ_0, σ_c, k, and of the variable Z. The fact that the lines 1 and 3' besides 3 and 1' intersect on P_ℓ' while the lines 1' and 2' as well as 1 and 2 and also 3 and 3' intersect on P_e' reduces from 12 to 6 the number of coordinates to be calculated.

The figure 7 shows how the elastic domain enlarges as Z increases, the lines forming the contour remaining parallel to themselves.

The graphic method described in the preceding paragraph is also valid in this case. It is slightly complicated by the fact that the dimensions of the elastic zone must be calculated as a function of Z, but on the other hand it is considerably simplified by the fact that the axes V and W are fixed relative to the axes P_i', P_ℓ', P_e'.

3 - RELATIONS OF ELASTIC LOADING CORRESPONDING TO THE CRITERIUM OF TRESCA [5]

Letting $\sigma_0 = \sigma_c$ the lines D of the plane τ, σ become parallel and the criterium of Mohr-Caquot reduces to that of Tresca. The equations of the six preceding planes are now :

$$P_i \frac{1+k^2}{k^2-1} - P_e \frac{2k^2}{k^2-1} + P_\ell = \pm \sigma_0,$$

$$P_i - P_e = \pm \sigma_0 \frac{k^2-1}{2k^2},$$

$$-P_\ell + P_i = \pm \sigma_0.$$

In the space V, W, Z defined in the preceding section the equations of these planes are reduced to :

1) 1') $$V - \frac{1}{\sqrt{3}}\,\frac{3k^2+1}{k^2-1}\,W = \pm\sqrt{\frac{2}{3}}\,\sigma_0,$$

2) 2') $$W = \mp\frac{\sqrt{2}}{4}\,\frac{k^2-1}{k^2}\,\sigma_0,$$

3) 3') $$V + \frac{1}{\sqrt{3}}\,W = \mp\sqrt{\frac{2}{3}}\,\sigma_0 .$$

Since the coordinate Z no longer appears in these expressions, these planes are normal to the plane π . They form an irregular hexagonal prism inscribed in the elliptic cylinder of Mises. The magnitude of the elastic domain is once again, as in the case of Mises, independant of the hydrostatic component of the load vector. The intersection of this prism with the plane π gives the contour of the elastic domain. The figure 8 shows that this contour can be traced in a remarkably simple way, the other half of the hexagon being symmetric with respect to the origine.

For k = 3, side 1 of the hexagon almost coincides with the perpendicular to p_e' (the position of 1 in the figure corresponding to k = ∞). This shows that increasing k above the value 3 adds only a very small gain to the elastic loading. For k = 1, the hexagon is reduced to the line p_ℓ' (zero surface).

The study of the maxima allows one to rediscover the following well-known result : for a given p_e there exists, contrary to the criterium of Mises an infinite number of values of p_ℓ for which p_i is maximum and equal to $\frac{\sigma_0}{2}\left(1-\frac{1}{k^2}\right)$. This results from the fact that the tangent of the contour drawn parallel to p_ℓ' coincides with side 2 of the hexagon. The extremities of the load vectors corresponding to the cases of cylinders open, closed, and in plane strain condition end respectively at the points a, b, and c of the figure 8.

4 - CONDITIONS OF APPLICATION AND LIMITS OF VALIDITY OF THESE METHODS

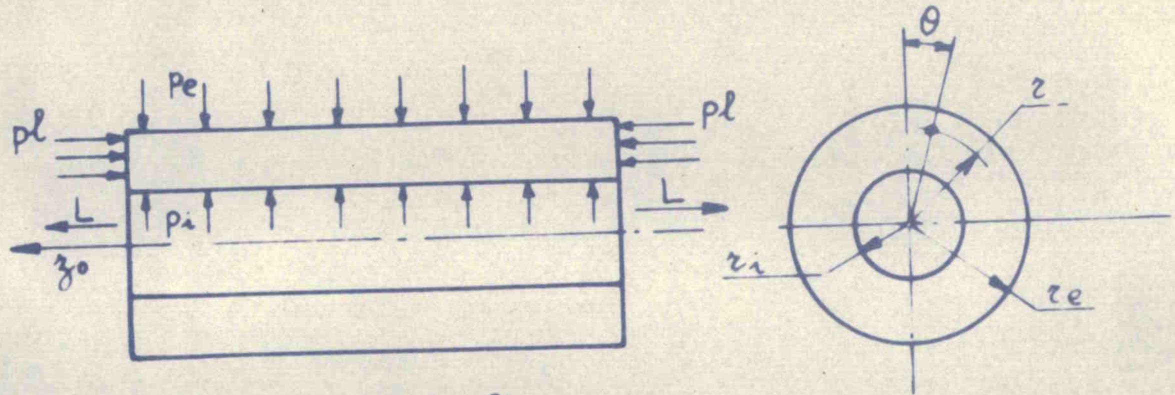
In the use of these methods we have implicitly assumed that the loads increase proportional to the same parameter. This condition is automatically satisfied for the cases of open and closed cylinders as well as for a cylinder in plane strain condition. However, it is no longer true for the case of shrink fits, where one applies first p_e followed then by p_i. The method presented here is nevertheless always valid on the condition that it is utilized in two steps.

In that which concerns the limits of validity of these methods we can make the following remarks. For a hydrostatic load $p_R = p_i = p_\ell = p_e$, of large magnitude the relation stress-strain should no longer be linear, in other words, Hookes law should cease to be valid. The work of Bridgman [6] on the compressibility of pure iron shows that already at 12,000 Atmospheres there exists small divergences from linearity.

Further if the deformations become large the relations between the components of the deformation tensor and the spatial derivatives of the components of the displacements become quadratic. At this point, the Lamé equations which are formed from the linear forms at these relations are no longer valid, and the relations of elastic loading, which are derived from them, must be entirely reconsidered. Thus, even, if the criterium of plasticity used, as in the case for the criteria of Mises, Mohr-Caquot and Tresca, implies the condition that a hydrostatic constraint does not cause plastic deformation, it does not automatically result from this that a hydrostatic load $p_R = p_i = p_\ell = p_e$ protects the cylinder from all plastic flow.

On the other hand, if the loads though very large are not isotropic ($p_i \neq p_\ell \neq p_e$) one can think that a plastic law governs the deformation beyond the elastic regime of Hookes law. On other words, there is no phase of a non linear law elasticity and consequently, the relations of elastic loading should remain valid.

(6) The Physics of High Pressure Bridgman P.W. G. Bell & Sons London 2nd Ed. 1949 p. 154.



$$L = -\pi(r_e^2 - r_i^2)p_\ell$$

Fig. 1

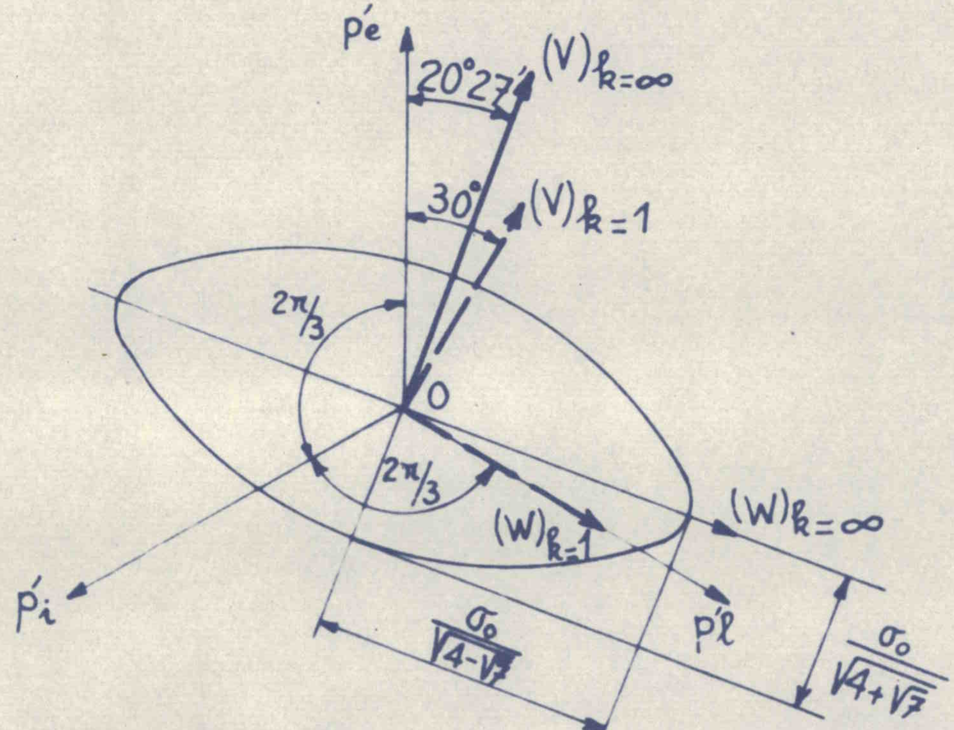


Fig. 2

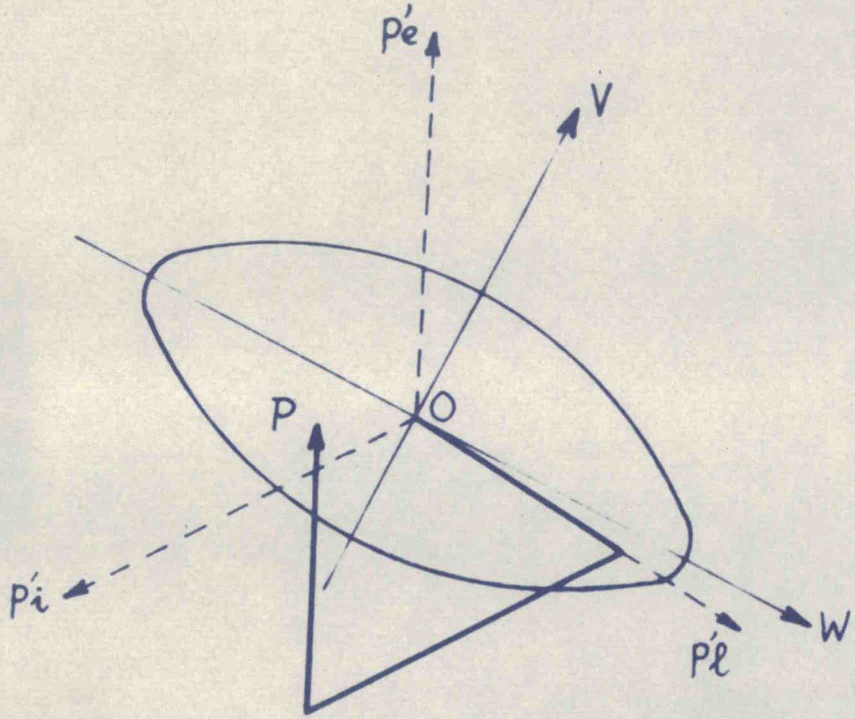


Fig 3

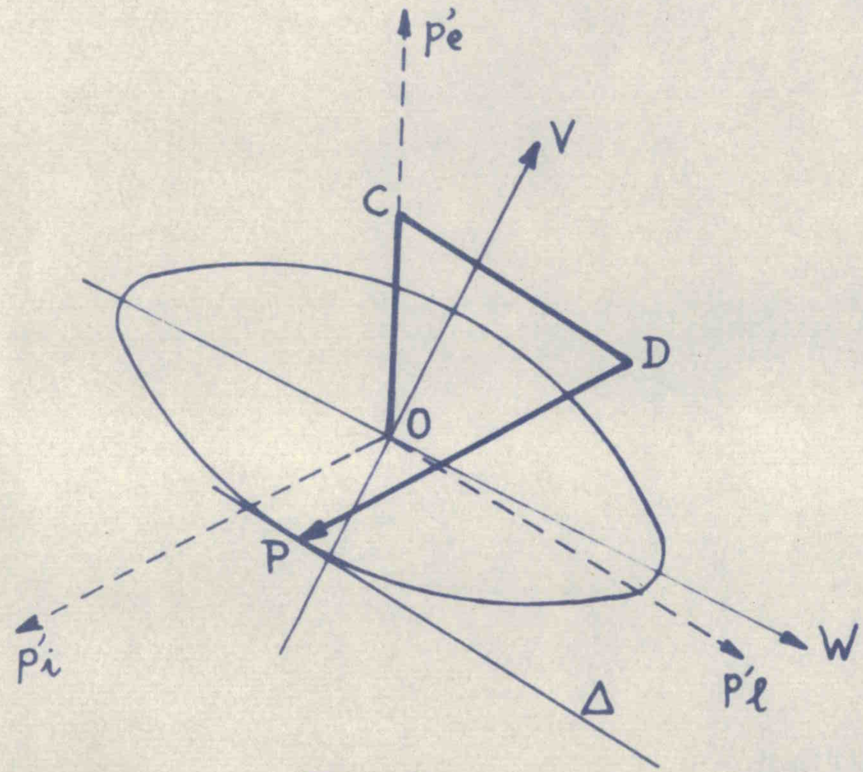


Fig. 4

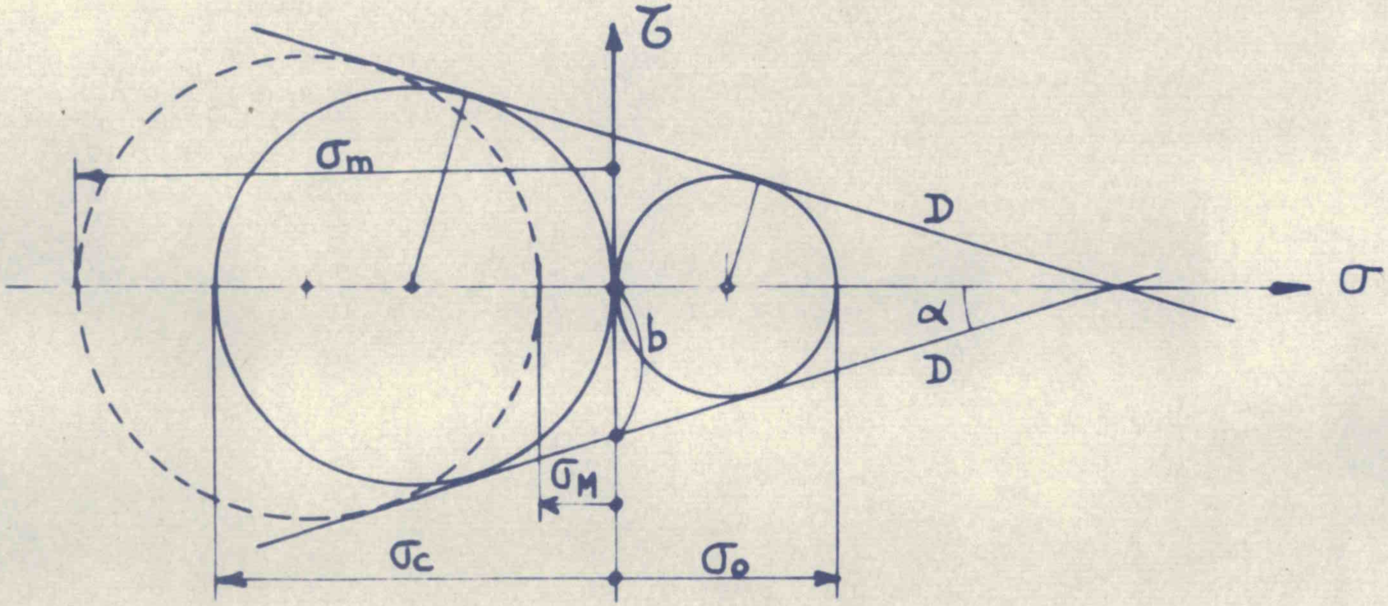


Fig. 5

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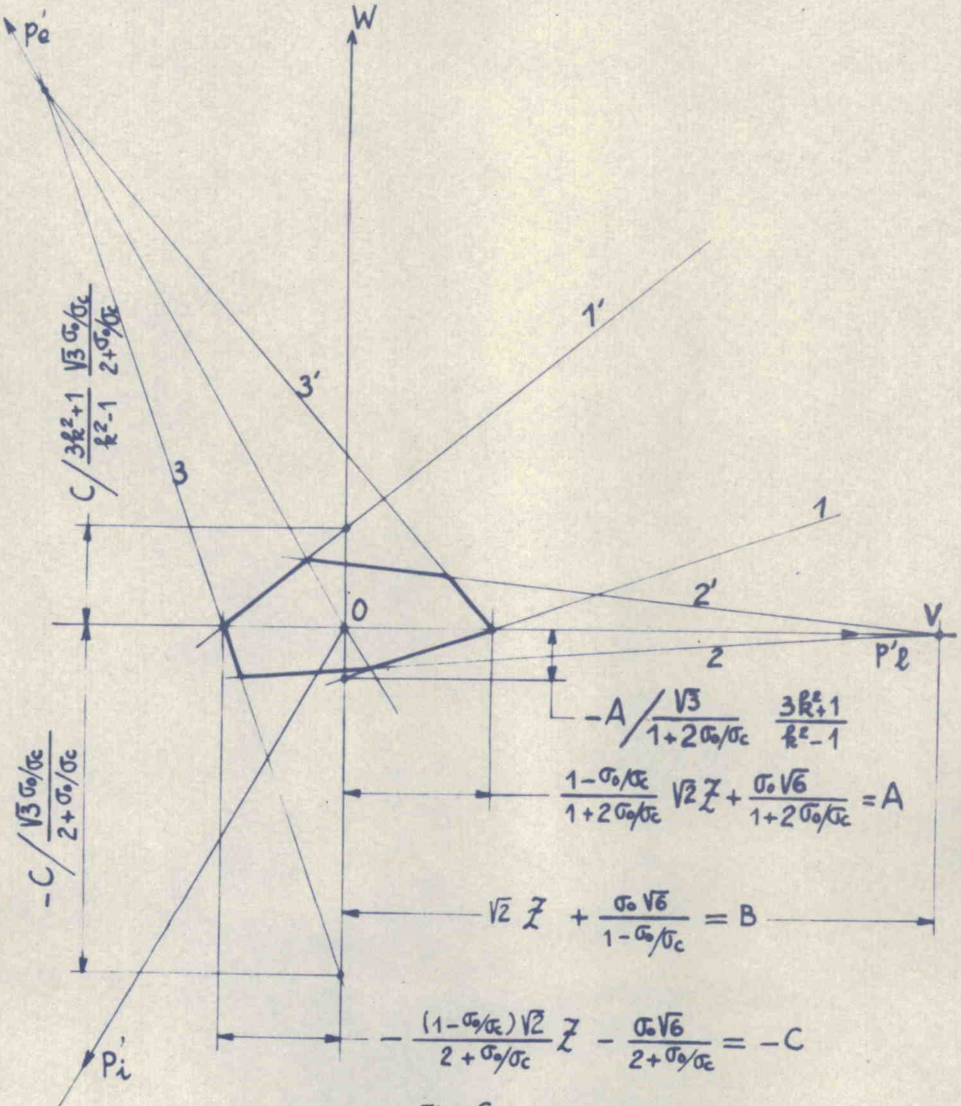

Fig. 6

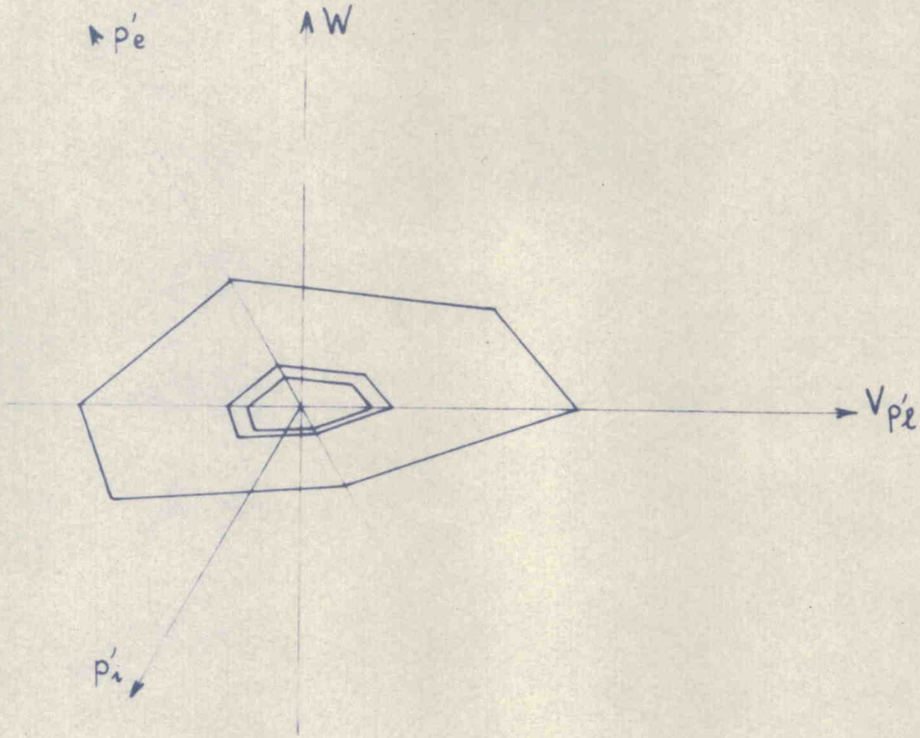

Fig 7

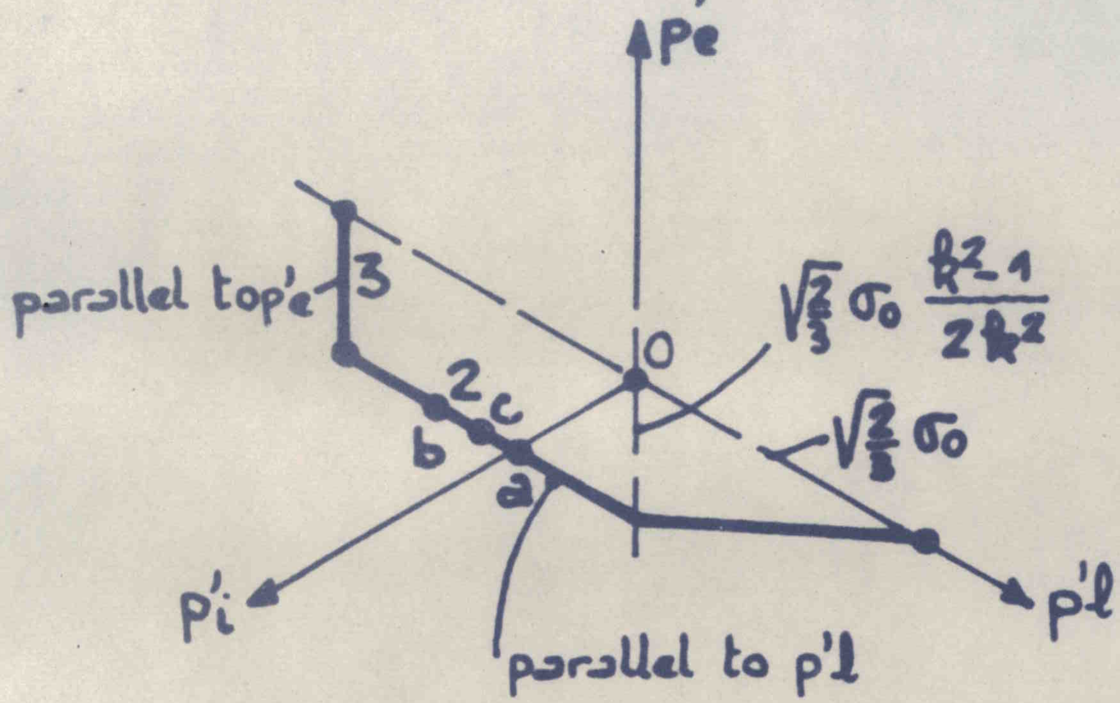
p'_e
0
$\sqrt{\frac{2}{3}}\,\sigma_0\,\frac{k^2-1}{2k^2}$
$\sqrt{\frac{2}{3}}\,\sigma_0$
parallel to p'_e
3
2
c
b
a
p'_i
p'_l
parallel to p'_l

Fig. 8